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# The effect of the quantum nature of excitons on the phonoriton states in semiconductors

Nguyen Minh Khue, Nguyen Que Huong and Nguyen Hong Quang  
Institute of Theoretical Physics, National Centre for Natural Science and Technology of  
Vietnam, PO Box 429 Bo Ho, Hanoi 10000, Vietnam

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**Abstract.** A treatment for the phonoriton problem based on the model introduced by Wang and Birman is proposed. It is shown that the model describes an interacting phonoriton system rather than a free one. The phonoriton spectrum is then calculated by using the Matsubara Green function technique. It is seen from the result that the phonoriton–phonoriton interaction narrows the gap in comparison with the gap in the spectrum of free phonoritons obtained by Wang and Birman. The damping of phonoriton states due to the interaction is explicitly calculated as well.

## 1. Introduction

Although excitons, photons and phonons are well known elementary excitations in semiconductors, to date the properties of a system of the mentioned interacting excitations have still attracted great attention, experimentally as well as theoretically. In 1982 Ivanov and Keldysh [1] suggested that if the crystal is excited by a strong monochromatic beam of light then the exciton–phonon interaction should lead to the formation of a new elementary excitation called a phonoriton. The mechanism of the formation of phonoritons and their properties have been discussed in many works by the Keldysh group [1–5]. Although some experiments have been carried out to show the existence of phonoriton states through photoluminescence [6, 7] so far there has been no direct evidence on the existence of such states except two other experimental works relating to phonoritons implicitly [8, 9].

Theoretically, the physical picture of the formation of phonoritons has recently been clarified by Wang and Birman in a very simple way [10]. In order to understand the properties of phonoritons in more detail, however, further studies are required since in the above-mentioned work many approximations were used. One of them is the replacement of the creation and annihilation operators for a polariton at pump mode by their quantum average values. Such a semiclassical approximation may ignore many interesting properties of phonoritons, which relate to the quantum nature of the excitons.

In this work we propose a simple approach, which enables us to acquire some information omitted in the above-mentioned approximation. The paper is organized as follows. Section 2 is devoted to the description of the model. Our approach is developed in section 3. The spectrum and the damping of the phonoriton are calculated in section 4, based on the Hamiltonian obtained in section 3 by using the Matsubara Green function technique. Section 5 is devoted to discussion and the conclusion. We use the unit system with  $\hbar = c = 1$ .

## 2. The model

We consider a system of interacting excitons, photons and longitudinal phonons described by the following Hamiltonian [1]

$$H = \sum_p \left\{ \omega_p^{\text{ex}} a_p^+ a_p + \omega_p^\gamma b_p^+ b_p + i \frac{\Omega_c}{2} (a_p^+ b_p - a_p b_p^+) + \Omega_p c_p^+ c_p \right\} + \sum_{pq} iM(p-q) a_p^+ a_q (c_{p-q} - c_{-(p-q)}^+) \quad (1)$$

$$\omega_p^{\text{ex}} = \omega_0^{\text{ex}} + \frac{p^2}{2m^*} \quad \omega_p^\gamma = \frac{p}{\sqrt{\epsilon_b}} \quad (2)$$

where  $a_p, b_p, c_p$  ( $a_p^+, b_p^+, c_p^+$ ) are annihilation (creation) operators for an exciton, a photon and a phonon with momentum  $p$ , respectively;  $\omega_p^{\text{ex}}, \omega_p^\gamma$  and  $\Omega_p$  are the energy of the exciton, photon and phonon, respectively. Here  $\omega_0^{\text{ex}}$  is the exciton energy at  $p = 0$ ,  $\epsilon_b$  is the background dielectric constant of the semiconductor,  $m^*$  is the effective mass of an exciton.  $\Omega_c$  is the photon–exciton interaction and  $M(p-q)$  is the matrix element of the exciton–phonon interaction.

Wang and Birman [10] showed that if a highly intense beam of light with frequency  $\omega_0$  enters the system and if the contributions of the two branches of polaritons can be considered separately (below we consider the lower branch) then the Hamiltonian given by [1] can be approximated by

$$H_{k_0} = \sum_p \omega_p^{\text{pol}} B_p^+ B_p + \sum_p \Omega_p c_p^+ c_p + H_S + H_{AS} \quad (3)$$

where

$$H_{AS} = i \sum_p \bar{M}(p-k_0) (B_{k_0} B_p^+ c_{p-k_0} - B_p B_{k_0}^+ c_{p-k_0}^+) \quad (4)$$

$$H_S = i \sum_p \bar{M}(p-k_0) (B_{k_0}^+ B_p c_{k_0-p} - B_p^+ B_{k_0} c_{k_0-p}^+)$$

in which  $B_p$  ( $B_p^+$ ) and  $\omega_p^{\text{pol}}$  are the annihilation (creation) operators and the energy of the polariton in the lower branch with momentum  $p$ , respectively.  $H_{AS}$  and  $H_S$  are called the anti-Stokes and Stokes scattering Hamiltonians.  $\bar{M}(p-k_0)$  is now the matrix element of the polariton–phonon interaction.

In order to solve the problem with Hamiltonian (3) Wang and Birman replaced  $B_{k_0}^+$  and  $B_{k_0}$  by their quantum average values in a coherent state of polariton mode  $k_0$ . The quantum nature of such polaritons is, therefore, omitted in the consideration. In the next section we shall develop a simple approach that enables us to understand the effect of the quantum nature of the excitons in mode  $k_0$ .

## 3. Phonoriton–phonoriton interaction

Instead of replacing  $B_{k_0}^+$  and  $B_{k_0}$  in  $H_{AS}$  and  $H_S$  simply by their quantum averages  $\langle B_{k_0}^+ \rangle_{\text{coh}}$  and  $\langle B_{k_0} \rangle_{\text{coh}}$  in coherent states we use the following identities:

$$B_{k_0}^+ = \langle B_{k_0}^+ \rangle + B_{k_0}^+ - \langle B_{k_0}^+ \rangle \quad B_{k_0} = \langle B_{k_0} \rangle + B_{k_0} - \langle B_{k_0} \rangle \quad (5)$$

where (...) represents the thermodynamic average.

Substituting (5) into (4) we have

$$H_{AS} = H_{AS}^{(1)} + H_{AS}^{(2)} \quad H_S = H_S^{(1)} + H_S^{(2)} \quad (6)$$

where

$$\begin{aligned} H_{AS}^{(1)} &= i \sum_p \bar{M}(p - k_0) \left\{ \langle B_{k_0} \rangle B_p^+ c_{p-k_0} - \langle B_{k_0}^+ \rangle B_p c_{p-k_0}^+ \right\} \\ H_{AS}^{(2)} &= i \sum_p \bar{M}(p - k_0) \left\{ B_{k_0} B_p^+ c_{p-k_0} - B_p B_{k_0}^+ c_{p-k_0}^+ \right\} \\ &\quad + i \sum_p \bar{M}(p - k_0) \left\{ \langle B_{k_0}^+ \rangle B_p^+ c_{p-k_0}^+ - \langle B_{k_0} \rangle B_p^+ c_{p-k_0} \right\} \\ H_S^{(1)} &= i \sum_p \bar{M}(p - k_0) \left\{ \langle B_{k_0}^+ \rangle B_p c_{k_0-p} - B_p^+ \langle B_{k_0} \rangle c_{k_0-p}^+ \right\} \\ H_S^{(2)} &= i \sum_p \bar{M}(p - k_0) \left\{ B_{k_0}^+ B_p c_{k_0-p} - B_p^+ B_{k_0} c_{k_0-p}^+ \right\} \\ &\quad + i \sum_p \bar{M}(p - k_0) \left\{ B_p^+ \langle B_{k_0} \rangle c_{k_0-p}^+ - \langle B_{k_0}^+ \rangle B_p c_{k_0-p} \right\}. \end{aligned}$$

Using the same arguments as in [10] we may study the spectra around anti-Stokes and Stokes frequencies individually. For illustration we shall consider here only the anti-Stokes spectrum. Paying attention to the fact that the number  $N_{k_0}$  of polaritons of pump mode is assumed to be fixed we should introduce into consideration the following Hamiltonian:

$$\tilde{H}_{k_0} = H_{k_0}^{(0)} + H_{k_0}^{int} \quad (7)$$

where

$$H_{k_0}^{(0)} = \sum_p (\omega_p^{pol} - \bar{\mu} \delta_{p,k_0}) B_p^+ B_p + \sum_p \Omega_p c_p^+ c_p + \tilde{H}_{AS}^{(1)} \quad (8)$$

$$H_{k_0}^{int} = \tilde{H}_{AS}^{(2)} \quad (9)$$

where

$$\tilde{H}_{AS}^{(1)} = i \sum_p \bar{M}(p - k_0) \left\{ B_p^+ \langle B_{k_0} \rangle c_{p-k_0} - \langle B_{k_0}^+ \rangle B_p c_{p-k_0}^+ \right\} \quad (10)$$

$$\begin{aligned} \tilde{H}_{AS}^{(2)} &= i \sum_p \bar{M}(p - k_0) \left\{ B_{k_0} B_p^+ c_{p-k_0} - B_p B_{k_0}^+ c_{p-k_0}^+ \right\} \\ &\quad + i \sum_p \bar{M}(p - k_0) \left\{ \langle B_{k_0}^+ \rangle B_p c_{p-k_0}^+ - B_p^+ \langle B_{k_0} \rangle c_{p-k_0} \right\} \end{aligned} \quad (11)$$

and  $\bar{\mu}$  is the chemical potential and its value is determined by the following condition:

$$\langle B_{k_0}^+ B_{k_0} \rangle_{\bar{\mu}} = N_{k_0}.$$

The Hamiltonian  $H_{k_0}^{(0)}$  given by (8) can be easily diagonalized by a Bogoliubov transformation. Expressing  $B_p$  and  $c_{p-k_0}$  in terms of a new operators  $D_\mu(p)$  with  $\mu = 1, 2$  as follows:

$$B_p = \sum_{\mu} u_{\mu}(p) D_{\mu}(p) \quad c_{p-k_0} = \sum_{\mu} v_{\mu}(p) D_{\mu}(p) \tag{12}$$

with

$$v_{\mu}(p) = \left\{ \frac{W_{\mu}^{(0)}(p) - \omega_p^{\text{pol}} + \bar{\mu}}{2W_{\mu}^{(0)}(p) - \omega_p^{\text{pol}} + \bar{\mu} - \Omega_{p-k_0}} \right\}^{1/2}$$

$$u_{\mu}(p) = \frac{i\bar{M}(p - k_0)N_{k_0}^{1/2}}{W_{\mu}^{(0)}(p) - \omega_p^{\text{pol}} + \bar{\mu}} v_{\mu}(p) \tag{13}$$

where

$$W_{1,2}^{(0)}(p) = \frac{1}{2}(\omega_p^{\text{pol}} - \bar{\mu} + \Omega_{p-k_0}) \pm \frac{1}{2} \{ (\omega_p^{\text{pol}} - \bar{\mu} - \Omega_{p-k_0})^2 + 4\bar{M}^2(p - k_0)N_{k_0} \}^{1/2} \tag{14}$$

we obtain

$$H_{k_0}^{(0)} = \sum_{p,\mu=1,2} W_{\mu}^{(0)}(p) D_{\mu}^{+}(p) D_{\mu}(p) + (\omega_{k_0} - \bar{\mu}) B_{k_0}^{+} B_{k_0}. \tag{15}$$

Equation (15) enables us to explain  $D_{\mu}^{+}(p)$  and  $D_{\mu}(p)$  as the creation and annihilation operators for the phonoriton in branch  $\mu$  and with momentum  $p$ .

Substituting (12) into (11), we can express  $H_{k_0}^{\text{int}}$  in terms of phonoriton operators, as follows:

$$H_{k_0}^{\text{int}} = \sum_{p,\mu\nu} M_{\mu\nu}(p - k_0) N_{k_0}^{1/2} D_{\mu}(p) D_{\nu}^{+}(p) - \sum_{p,\mu\nu} M_{\mu\nu}(p - k_0) B_{k_0}^{+} D_{\mu}(p) D_{\nu}^{+}(p) + \text{HC} \tag{16}$$

where

$$M_{\mu\nu}(p - k_0) = i\bar{M}(p - k_0) u_{\mu}(p) v_{\nu}^{*}(p). \tag{17}$$

Equation (16) enables us to regard  $H_{k_0}^{\text{int}}$  as the Hamiltonian describing the phonoriton-phonoriton interaction and therefore we may say that the quantum nature of the excitons at mode  $k = k_0$  leads to the interaction between the phonoritons.

#### 4. The spectrum and damping of phonoritons

The interaction between phonoritons (16), as expected in any system with interaction, should give rise to the damping of phonoritons, as well as change in their spectrum. In order to calculate these we use the Matsubara Green function technique [11], namely we consider the following Green functions with imaginary time  $\tau$ :

$$F_{\mu\nu}(k, \tau) = -\langle T_{\tau} D_{\mu}^{+}(\tau) D_{\nu}(0) \rangle \quad G(k_0, \tau) = -\langle T_{\tau} B_{k_0}^{+}(\tau) B_{k_0}(0) \rangle. \tag{18}$$

The Fourier transformations of the corresponding free Green functions are

$$F_{\mu\nu}^{(0)}(k, i\omega_n) = \delta_{\mu\nu}(i\omega_n - W_{\mu}^{(0)}(k))^{-1} \quad G(k_0, i\omega_n) = (i\omega_n - \omega_{k_0}^{\text{ph}} + \bar{\mu})^{-1} \quad (19)$$

where  $\omega_n = \pi\beta^{-1}n$  with  $n$  being an even integer number,  $\beta = (1/k_B T)$  with  $k_B$  and  $T$  being the Boltzmann constant and temperature, respectively. The Fourier transformations of the Green function for the interacting phonoritons obey the following Dyson equation:

$$F_{\mu}(k, i\omega_n) \equiv F_{\mu\mu}(k, i\omega_n) = (i\omega_n - W_{\mu}^{(0)}(k) - \Sigma_{\mu}(k, i\omega_n))^{-1} \quad (20)$$

where  $\Sigma_{\mu}(k, i\omega_n)$  is the self-energy, and its diagrammatic expansion is of the following form:

$$\Sigma_{\mu}(k, i\omega_n) = \blacktriangle + \overset{\text{dashed}}{\text{---}} + \dots \quad (21)$$

in which the dashed and solid lines represent  $F^{(0)}(k, i\omega_n)$  and  $G(k_0, i\omega_n)$ , respectively. The vertex stands for the interaction strength given by (17).

In the lowest-order approximation the analytic expression corresponding to (21) is

$$\begin{aligned} \Sigma_{\mu}(k, i\omega_n) = & M_{\mu\mu}(k - k_0)N_{k_0}^{1/2} - \sum_{\nu_1 q_1} |M_{\mu\nu_1}(k - k_0)|^2 \{N(W_{\nu_1}^{(0)}(q_1)) \\ & + N(\omega_{k_0} - \bar{\mu})\} \Delta(q_1 - k) (i\omega_n - W_{\nu_1}^{(0)}(q_1) - \omega_{k_0} + \bar{\mu})^{-1} \end{aligned} \quad (22)$$

where

$$\begin{aligned} N(x) & \equiv [\exp(\beta x) - 1]^{-1} \\ \Delta(q_1 - k) & = \begin{cases} 1 & \text{for } q_1 = k \\ 0 & \text{for } q_1 \neq k. \end{cases} \end{aligned}$$

The retarded Green function corresponding to (20) can be obtained by the analytic continuation  $i\omega_n \rightarrow \omega + i\epsilon$ , where  $\epsilon$  is a positive infinitesimal number [11]. As a result we obtain

$$F_{\mu}(k, \omega) = (\omega - W_{\mu}^{(0)}(k) - \Sigma_{\mu}(k, \omega + i\epsilon) + i\epsilon)^{-1} \quad (23)$$

where

$$\begin{aligned} \Sigma_{\mu}(k, \omega + i\epsilon) = & M_{\mu\mu}(k - k_0)N_{k_0}^{1/2} - \sum_{\nu_1 q_1} |M_{\mu\nu_1}(k - k_0)|^2 \{N(W_{\nu_1}^{(0)}(q_1)) \\ & + N(\omega_{k_0} - \bar{\mu})\} \Delta(q_1 - k) (\omega - W_{\nu_1}^{(0)}(q_1) - \omega_{k_0} + \bar{\mu} + i\epsilon)^{-1}. \end{aligned} \quad (24)$$

To calculate the chemical potential constant  $\bar{\mu}$  in (24), as stated above, we have to know  $\langle B_{k_0}^+ B_{k_0} \rangle_{\bar{\mu}}$ . This quantity can be calculated through the retarded Green function  $G_r(k_0, \omega)$  of the polariton at mode  $k_0$ . It can be easily seen that if we approximate  $G_r(k_0, \omega) \simeq G_r^{(0)}(k_0, \omega)$  then at low temperature  $T \rightarrow 0$ , we have  $\bar{\mu} = \omega_{k_0}$ . Then the spectrum of the free phonoriton (14) coincides with the phonoriton spectrum (18) in [10].

From (23) the spectrum and damping of phonoritons can be obtained by solving the following non-linear equation:

$$\begin{aligned} \omega = W_{\mu}^{(0)}(k) + M_{\mu\mu}(k - k_0)N_{k_0}^{1/2} - P \sum_{\nu_1 q_1} |M_{\mu\nu_1}(k - k_0)|^2 \{N(W_{\nu_1}^{(0)}(q_1)) \\ + N_{k_0}\} (\omega - W_{\nu_1}^{(0)}(q_1))^{-1} \Delta(q_1 - k) + i\pi \sum_{\nu_1 q_1} |M_{\mu\nu_1}(k - k_0)|^2 \{N(W_{\nu_1}^{(0)}(q_1)) \\ + N_{k_0}\} \Delta(q_1 - k) \delta(\omega - W_{\nu_1}^{(0)}(q_1)) \end{aligned} \tag{25}$$

where the Dirac identity  $1/(x + i\epsilon) = P/x - i\pi\delta(x)$  has been used;  $P$  denotes the principal value of an integral under which this relation is used. Since the second, third and fourth terms in the right-hand side of (25) are small compared to the first one we can solve (25) by using the successive approximation method. To the first approximation we obtain the following expression for the phonoriton spectrum:

$$\begin{aligned} W_{\mu}(k) = W_{\mu}^{(0)}(k) + M_{\mu\mu}(k - k_0)N_{k_0}^{1/2} - |M_{\mu\nu}(k - k_0)|^2 \{N(W_{\nu}^{(0)}(k)) \\ + N_{k_0}\} (W_{\mu}^{(0)}(k) - W_{\nu}^{(0)}(k))^{-1} \end{aligned} \tag{26}$$

and for the damping

$$\Gamma_{\mu}(k) = \frac{\pi V}{(2\pi)^3} |M_{\mu\mu}(k)|^2 \{N(W_{\mu}^{(0)}(k) + N_{k_0}) f_{\mu}(k) k^2 \tag{27}$$

where  $\nu \neq \mu$ , and

$$f_{\mu}(k) = [\partial W_{\mu}^{(0)}(q) / \partial q |_{q=k}]^{-1}. \tag{28}$$

At low temperatures and for a sufficiently intense pump field we have

$$N(W_{1,2}^{(0)}(k)) \ll N_{k_0}. \tag{29}$$

Substituting (17) into (26) and (27) we obtain the following final result for the spectrum and the damping of the phonoriton:

$$\begin{aligned} W_1(k) = W_1^{(0)}(k) - \frac{\bar{M}^2(k - k_0)N_{k_0}}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}^{1/2}} \\ + \frac{\bar{M}^4(k - k_0)N_{k_0}^2(W_2^{(0)} - \omega_k^{\text{pol}} + \omega_{k_0})}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_0\}^{3/2}(W_1^{(0)} - \omega_k^{\text{pol}} + \omega_{k_0})} \end{aligned} \tag{30}$$

$$\begin{aligned} W_2(k) = W_2^{(0)}(k) + \frac{\bar{M}^2(k - k_0)N_{k_0}}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}^{1/2}} \\ + \frac{\bar{M}^4(k - k_0)N_{k_0}^2(W_1^{(0)} - \omega_k^{\text{pol}} + \omega_{k_0})}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_0\}^{3/2}(W_2^{(0)} - \omega_k^{\text{pol}} + \omega_{k_0})} \end{aligned}$$

$$\Gamma_1(k) = \frac{\pi V}{(2\pi)^3} \frac{\bar{M}^4(k - k_0)N_{k_0}^2(W_1^{(0)}(k) - \Omega_{k-k_0})^{-1} k^2}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}^{1/2} V_k^{\text{pol}}} \frac{1}{V_k^{\text{pol}}} \tag{31}$$

$$\Gamma_2(k) = \frac{\pi V}{(2\pi)^3} \frac{\bar{M}^4(k - k_0)N_{k_0}^2(-W_2^{(0)}(k) + \Omega_{k-k_0})^{-1} k^2}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}^{1/2} V_k^{\text{pol}}} \frac{1}{V_k^{\text{pol}}}$$

where  $V_k^{\text{pol}}$  is the group velocity of the polariton

$$V_k^{\text{pol}} = \frac{1}{2} \left( \frac{k}{m^*} + \frac{1}{\sqrt{\epsilon_b}} \right) - \frac{1}{2} \frac{(k/m^* - 1/\sqrt{\epsilon_b})(\omega_k^{\text{ex}} - \omega_k^{\text{y}})}{\{(\omega_k^{\text{ex}} - \omega_k^{\text{y}})^2 + \Omega_c^2\}^{1/2}}. \tag{32}$$

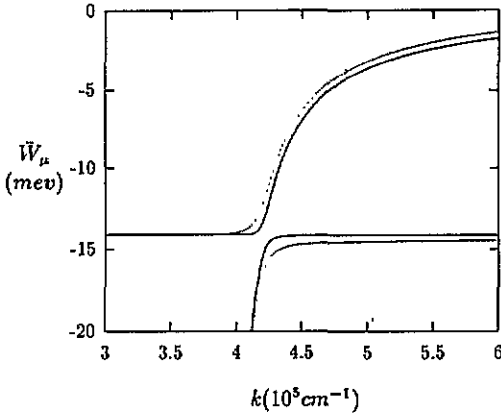


Figure 1. The phonoriton dispersion curve in CdS at pump frequency  $\omega_0 = 2500$  meV,  $\bar{W}_\mu = W_\mu - \omega_0^{\text{ex}}$ : solid line, our work; dotted line, [10]

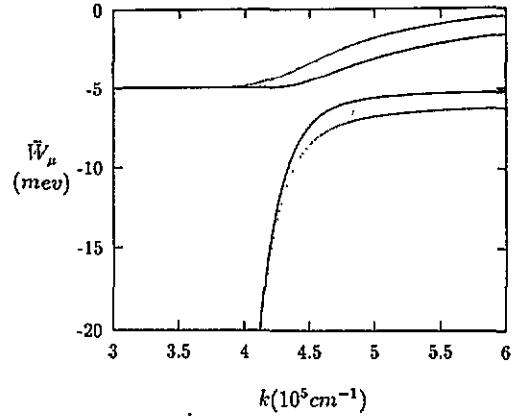


Figure 2. The phonoriton dispersion curve in CdS at pump frequency  $\omega_0 = 2510$  meV,  $\bar{W}_\mu = W_\mu - \omega_0^{\text{ex}}$ : solid line, our work; dotted line, [10]

## 5. Conclusion

As noted above, the phonoriton spectrum obtained in [10] is simply the spectrum of the free phonoriton (14) in our treatment. From (30) one can see that due to the phonoriton-phonoriton interaction the upper phonoriton branch goes down while the lower one shifts up in comparison with the spectrum of the free phonoriton in [10]. The gap between the two branches given by (30) is therefore smaller than that of free phonoritons. Namely, denoting

$$\Delta W(k) = W_1(k) - W_2(k) \quad \Delta W^{(0)}(k) = W_1^{(0)}(k) - W_2^{(0)}(k)$$

we have

$$\begin{aligned} \Delta W(k) = \Delta W^{(0)}(k) & - \frac{2\bar{M}^2(k - k_0)N_{k_0}}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}^{1/2}} \\ & + \frac{\bar{M}^2(k - k_0)N_{k_0}(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})}{\{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}\}} \end{aligned} \quad (33)$$

where

$$\Delta W^{(0)}(k) = \sqrt{(\omega_k^{\text{pol}} - \omega_{k_0} - \Omega_{k-k_0})^2 + 4\bar{M}^2(k - k_0)N_{k_0}}$$

as can be seen directly from (14).

At the resonance, when  $\omega_k^{\text{pol}} - \omega_{k_0} = \Omega_{k-k_0}$  we have  $\Delta W^{(0)}(k) = 2\bar{M}(k - k_0) N_{k_0}^{1/2}$  and  $\Delta W(k) = \bar{M}(k - k_0)N_{k_0}^{1/2}$ , i.e.,  $\Delta W(k) = \frac{1}{2}\Delta W^{(0)}(k)$ .

In figures 1-4 we have plotted the phonoriton spectrum and its damping in CdS at the frequencies of a pump field near the exciton resonance  $\omega_0 = 2500$  meV and 2510 meV, respectively. The data used for calculation are adopted from [10]:  $N_{k_0} = 10^{20} \text{ cm}^{-3}$ ,  $\bar{W}_\mu = W_\mu - \omega_0^{\text{ex}}$ ,  $\bar{\Gamma}_\mu = \Gamma_\mu / \omega_0^{\text{ex}}$ . For comparison, in figures 1 and 2, besides our curves (solid lines) we have plotted the phonoriton dispersion calculated by using the approximation



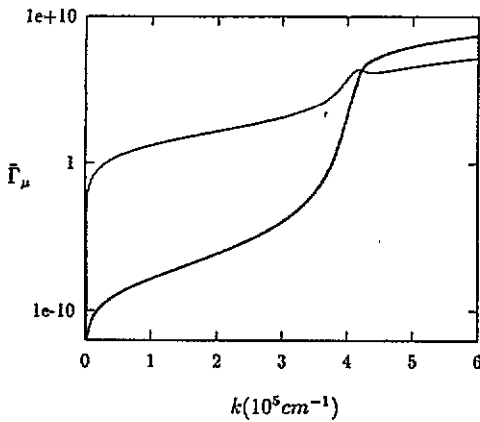


Figure 3. The damping of phonoritons in CdS due to the phonoriton-phonoriton interaction at pump frequency  $\omega_0 = 2500$  meV,  $\bar{\Gamma}_\mu = \Gamma_\mu/\omega_0^\alpha$ .

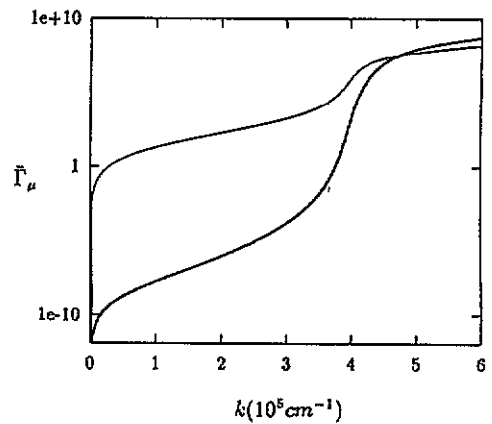


Figure 4. The damping of phonoritons in CdS due to the phonoriton-phonoriton interaction at pump frequency  $\omega_0 = 2510$  meV,  $\bar{\Gamma}_\mu = \Gamma_\mu/\omega_0^\alpha$ .

in [10] (dotted lines). It is clearly seen from these figures that the interactions between phonoritons have quite an important influence on their spectrum, narrowing the gap of the phonoritons. Perhaps to a certain extent this explains the difficulty in finding phonoritons experimentally.

Finally we would like to note that it is the interaction between phonoritons that leads to the damping of the phonoritons. The phonoriton state in [10] is only a free one in our treatment, therefore it will have no damping unless one includes it phenomenologically.

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